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What can we learn from how gifted/average pupils describe their processes of problem solving?

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Abstract

Research on the cognitive sub-processes involved in the excellent problem solving of the gifted, as compared to the problem solving of the average person, has attributed the difference between these two populations to selectivity in their Encoding, Comparison and Combination sub-processes. This paper extends this list by adding two sub-processes that are imported from the literature on experts and novices: namely, Retrieval and Goal Directness. Based on these five sub-processes in conjunction with the concept of selectivity as an ordinal (rather than dichotomous) dimension, we have constructed a model that is being used for the analysis of the solution processes of gifted and average students, as reflected in their post solution protocols. Middle high school students (gifted and average) solved insight problems, without and with analogical learning, and were asked to report on the solution process they undertook. The suggested model was found to be an effective instrument for analyzing the sub-processes employed during problem solving. Though both the gifted and the average were able to arrive at correct solutions, the study shows that they employed different sub-processes in doing so. The model can serve as a fine-grade analysis of solution processes among various populations (gifted/average and possibly experts/novices) that will be helpful in research and teaching.

Keywords: Problem solving process; Gifted; Analogical learning; Insight problems

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1. Introduction

By definition, the gifted are excellent problem solvers. This ability distinguishes them from the rest of the population in terms of scores on I.Q. tests that mainly examine problem-solving capabilities. Studies have attempted to reveal the unique cognitive processes used by this population that lead to these strikingly different results. Indeed, Sternberg and Davidson (1983) have identified three major cognitive components; Encoding, Comparison and Combination, that were responsible for the correct solutions of insight problems by gifted. Another population of successful problem solvers is that of experts, as opposed to novices. For this population, characterized by expertise in the solution of domain-specific problems, two additional cognitive sub-processes have often been attributed to their solution processes: Retrieval and Goal Directness (for research on this population, see, e.g., Berger & Wilde, 1988; Chi, Feltovich, & Glaser, et al., 1981; Gobbo & Chi, 1986; Low & Over, 1992; Reed, Willis, & Guarino, 1994; Resnick 1985). Indeed, the inevitable association between gifted and experts, and between average and novices, has already led to suggestions to integrate the two streams of research (Rabinowitz & Glaser, 1986). Lately, Sternberg (1998) joined this trend, claiming that the achievements and abilities (of experts and of gifted) reflect two kinds of expertise: expertise in a specific domain and expertise in solving problems in the general domain (insight problems). Notwithstanding the different nature of the problems and/or the nature of the solvers, the cognitive processes that have been mapped seem to be similar (Shore & Kanevsky, 1993).

In this paper we propose to incorporate these five sub-processes, via a mapping sentence, into a model for the analysis of the reflections of gifted and average students on their solution processes. This analysis concentrates on qualitative statements regarding the solution process, rather than on quantitative achievements (correct/erroneous). We do not attempt to claim that the proposed model is a cognitive model for problem solving. Rather, we wish to use it to typify the differences between what gifted and average students do during the solution process, as evidenced from their protocols.

Each sub-process included in the model is measured along the common dimension of selectivity, which is indicative of the sub-process efficiency and sophistication. Thus, a solution profile can be obtained for each solver in terms of the selectivity level employed for each sub-process. Before introducing the model, we will clarify the nature of the various cognitive sub-processes that are included, and the nature of the dimension of selectivity that cuts across all sub-processes.

1.1. Selectivity

Selectivity has been found to be a dimension that distinguishes between the gifted and the average, in terms of the effectiveness of cognitive sub-processes of problem solving (Sternberg & Davidson, 1983; Davidson, 1986). In this earlier research, it was applied as a dichotomous concept, distinguishing between effective sub-processes of the gifted that led to correct solutions and ineffective sub-processes that led average solvers to erroneous solutions. In the present paper selectivity is used as an ordinal dimension that can be applied to each detected sub-process in students' solutions, regardless of success or failure.

1.2. Encoding

Encoding refers to the sub-process by which the solver extracts information from a given problem. The gifted were found to exhibit more selective encoding than average people, and this selective encoding was attributed to their general knowledge and intelligence (Davidson & Sternberg, 1984; Sternberg & Davidson, 1983). Davidson (1986) found that the gifted spontaneously use analogies (deep structure) to encode relevant information, whereas the average depend on explicit cues (surface structure) for finding essential information for the solution. This finding links the encoding of deep structure information with a more efficient and sophisticated solution process.

Similarly, experts have been found to encode detailed information, as well as the implicit deep-structure relations of the problem, and to ignore irrelevant features embedded in the problem. Novices, in contrast, tend to concentrate on the surface structure, i.e., on literal pieces of information which may be irrelevant for the solution (Chi, Feltovich, & Glaser, 1981; Gobbo & Chi, 1986; Mayer, 1982; Resnick, 1985). This mode of encoding by experts has been attributed to their knowledge structures, which guide them to concentrate on relevant and important information (Low & Over, 1992).

In keeping with the above findings, we use three levels of selectivity to analyze the sub-process of encoding. Deep-structure encoding refers to the most selective level, followed by a combination of deep and surface structure, and finally by surface structure alone (least selective).

1.3. Combination

In this sub-process, the solver combines encoded information, its semantic interpretation and retrieved procedural solution knowledge into a solution structure. In a study of insight — domain free problems that compared gifted and average subjects, Sternberg and Davidson (1983) distinguished between selective and non-selective combinations (a combination was considered selective if it combined pieces of information into an integrative solution process). They found the combination process of the gifted to be more selective than that of the average population. Similarly, Coleman and Shore (1991) report that the gifted (based on achievements in physics) used a higher abundance of statements that indicated integration with prior knowledge, whereas the average focused mainly on replication of the information presented in the problem, failing to integrate it with relevant prior knowledge.

This sub-process of combination is similar to the sub-process of integration analyzed by Berger and Wilde (1988) in their studies on experts and novices in mathematics. These researchers suggest three hierarchical levels of knowledge integration, where "combination" is the highest level (denoting the integration into a unified solution structure of the encoded and retrieved concepts, the relations, and procedures associated with the deep structure of the problem).

Peled and Wittrock (1990) provide yet another hierarchy for knowledge combination. In their research, which deals with mathematical problems, integration is considered the highest level of knowledge combination (i.e., the most selective). There are two less efficient combinations: a replicative one which reflects a direct translation of the text into an equation, without introducing any changes; and a distortive combination which is performed on limited or erroneous knowledge, using non-specific strategies. In the present paper, we adopt the three categories used by Peled and Wittrock: integrative, replicative and distortive combinations.

1.4. Comparison

The sub-process of comparison (Gentner & Markman, 1997; Sternberg & Davidson, 1983) — also called analogical reasoning (Mayer, 1992; Kolodner, 1997) or analogical transfer (e.g., Gick, & Holyoak, 1980, 1986; Holyoak & Koh (1987) — refers to the solver's search for a pattern that may lead to a solution, and concurrent comparison of that pattern with possible solution structures attained in past learning. This cognitive sub-process is fundamental in the more common problem-solving situations, e.g., school and everyday life, where the solution of new problems often derives from previous experience and learning of analogous problems.

Researchers claim that high intelligence and giftedness are associated with selective (i.e., effective) comparison (Davidson & Sternberg, 1984; Sternberg & Davidson, 1983) or analogical reasoning (Gentner & Markman, 1997; Zook, 1993; Zook & Maier, 1994). Davidson (1986) found that the gifted used knowledge from source problems more effectively for the solution of target problems that appeared to be different on the surface but were equivalent in their deep structure. A more recent study found that worked-out source problems in either a visual-humorous or a verbal modality aided students, gifted and average alike, in solving analogical target problems in the complementary modality (Klavir & Gorodetsky, 2001). In this sub-process, solvers have to systematically map the deep-structure features and their relations between source and target problems, while ignoring irrelevant information included in the problem (Chi, Feltovich & Glaser, 1981; Gentner & Holyoak, 1997; Reed, Willis, & Guarino, 1994; Zook & Maier, 1994). Experts were found to draw this ability from their vast experience and their tendency to view each problem as a specific case of a general solution structure (Resnick, 1985; Kolodner, 1997). Novices, on the other hand, were distracted by the content of the problem, which was just as likely to lead them to a suitable source problem for comparison, as to unsuitable procedures for the solution (Reed, Willis, & Guarino, 1994).

In this study, we distinguish between four levels of selectivity in the comparison sub-process. In the most selective level, only deep-structure relations are compared. Less selective levels of comparison (from greater to lesser selectivity) are: partial deep-structural relations, surface-structure relations, and no comparison of relations.

1.5. Retrieval

Retrieval refers to the activation of concepts and terms that enable the interpretation of a given problem in the solver's terms. While Coleman and Shore (1991) used the term to signify all kinds of knowledge retrieved for the interpretation of the problem, Fan et al. (1994) distinguished between retrieval of terms that enable interpretation of the text, and retrieval of procedural knowledge for a solution. We, too, feel it is important to differentiate between the two kinds of retrieved knowledge. Thus, we ascribe retrieval only to cases of semantic knowledge including semantic aspects of the text (i.e., terms that refer either to the surface or deep structure), and we, rather, include retrieval of knowledge related to the solution structure under the sub-process of Comparison (see above).

Experts and high achievers seem to reorganize new problems into familiar terms automatically. This process is mainly associated with the deep structure that is necessary for the solution. Novices, in contrast, tend to adhere to the new concepts and propositions exactly as stated in the new problem, or to retrieve extraneous information that is unnecessary for the solution (Gobbo & Chi, 1986; Lawson and Chinnappan, 1994). Research on the gifted vs the average has yielded similar results (Coleman and Shore, 1991). However, one should bear in mind that the gifted in the Coleman & Shore research were defined on the basis of achievements in physics, and thus may be considered as relative experts, and the authors did not distinguish between retrieval of semantic terms and knowledge related to the solution structure.

1.6. Goal Directness

Experts are faster than novices in reaching solutions (Simon & Simon, 1978) and their solution processes tend to be goal-directed (Gick & Holyoak, 1980; Novick and Holyoak, 1991). Their search is schema driven (Gilhooly, 1996) chunking the information into large schemata and avoiding being distracted by detailed information. In cases of complex problems for which they lack a suitable solution schema, their search for a solution is systematic. This search is influenced by "self management," "executive" or "meta-cognitive" skills that direct the process (Schoenfeld, 1983). The process is goal directed and is accompanied by "reflection in action," i.e., continuous self-regulation and tuning (Jones & Idol, 1990).

In contrast, the novice's solution process seems to lack monitoring (Gilhooly, 1996). It is impulsive and less controlled (Glaser, 1984), and, even when it seems to be goal directed, it lacks self-corrective processes (Lawson & Chinnappan, 1994).

Likewise the experts, the solution processes of gifted are faster than those of the average — a feature that is conveyed in intelligence tests (Davidson & Sternberg, 1984). The gifted are also capable of monitoring their solution pathway and moving from one pathway to another (e.g., Tarshis & Shore, 1991; Dover & Shore, 1991). In the present study, we distinguish between three selectivity levels of goal directness: problem solving that is directed to the final goal, one that entails a systematic search and one that involves a random search.

2. The model

The proposed model has been formulated as a mapping sentence (Fig. 1), i.e., a semantic frame for describing observed information. The sentence is made up of six facets, five of which represent sub-processes of the solution and the last referring to the correctness of the solution. For each sub-process, there is a range of descriptive categories (levels of selectivity), based on the literature described above. These categories are ordinal, ranging from the most to the least selective.

The mapping sentence enables analysis of the entire reported solution process, as well as of each separate sub-process, and it allows us to obtain a solution profile for each solver. The Most Selective Profile (MSP) was defined as one in which the solver *encodes deep-structure items, retrieves deep-structure information* relevant to the interpretation of the problem, performs an *integrative combination* in a process that is *directed to the final goal*, and reports on the *comparison of only deep-structure relations* with an analogical problem from past learning.

The specific problems chosen for this research were insight problems found to distinguish between gifted and average populations (e.g., Davidson, 1986). They were presented in both a visual-humorous modality (cartoons) and in a verbal modality (word problems). In order to ensure use of all sub-processes, including comparison, analogical transfer was monitored. That is, analogical learning through worked-out source problems was compared to a situation without prior learning.

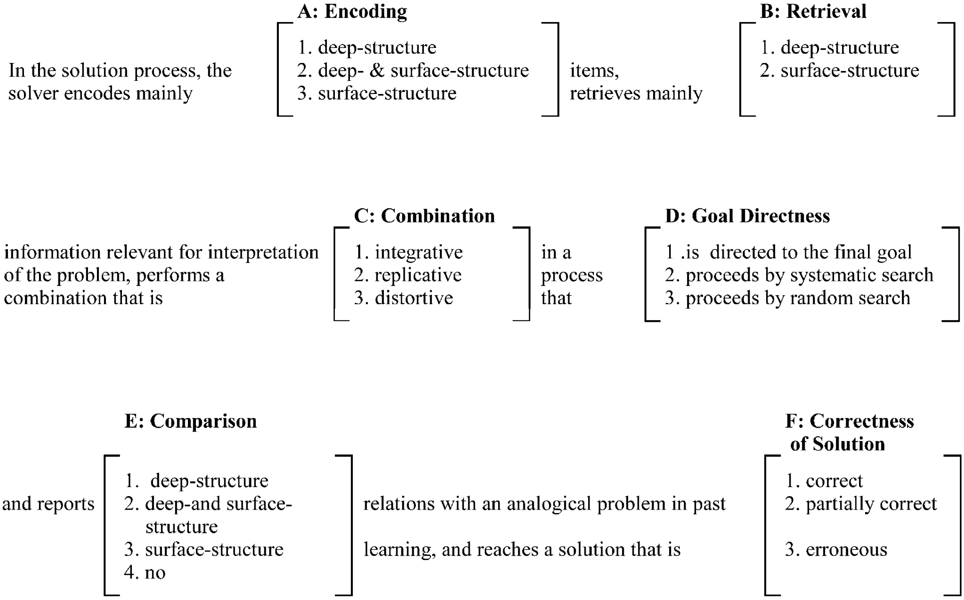

Fig. 1. Mapping sentence

3. Method

3.1. Subjects

The research was carried out among 121 gifted and average students, from an urban, middle-class background, attending the same comprehensive school. The gifted subjects were seventh and eighth graders with an I.Q. defined by the Ministry of Education as above 131 (*N*=60); they studied in special classes for the gifted. The average subjects were eighth and ninth graders defined as neither gifted nor as requiring special education (*N*=61); they studied in regular classes. The difference in the ages of the populations was to ensure similar mental age (Planche, 1985).

3.2. Materials

Four main questionnaires were used: two that examined problem solving without prior learning (either in the visual-humorous or verbal modality) and two that reflected the transfer of solutions from one modality to the other, after learning from worked-out analogous examples. The analogical nature of the source and target problems in the two modalities has been validated (Klavir & Gorodetsky, 2001). The sequence of the problems in each questionnaire was random.

Each student was given only one questionnaire out of each pair (1 or 2 and 3 or 4). Questionnaires 1 & 2 comprised the situation without prior learning, and questionnaires 3 & 4 comprised the situation of learning from analogical worked-out problems, as follows:

Without prior learning		Learning from worked-out analogous examples	
Questionnaire no. 1 Included two problems to be solved in the visual-humorous modality (cartoons).	Questionnaire no. 2 Included two problems to be solved in the verbal modality.	Questionnaire no. 3 Included two solved problems in the visual-humorous modality (cartoons), followed by two analogous verbal problems to be solved by the students.	Questionnaire no. 4 Included two solved problems in the verbal modality, followed by two analogous visual-humorous problems to be solved by the students.

3.2.1. The problems:

To gain information on the sub-process of Comparison and its dependence on analogical learning, by both gifted and average populations, we have used in the research analogical problems. The source and target problems were identical in their deep structure and solution process and differed in all other aspects, namely in their content (story), context and modality (verbal/visual). Extending the differences between the source and target analogical problems is shielding the immediate recognition of the resemblance (Holyoak & Koh, 1987; Catrambone & Holyoak, 1989; Gentner & Toupin, 1986; Okagaki & Koslowski, 1987). Thus the larger the differentiation in the surface structure between the source and target problems, it will be a more sensitive tool to differentiate between the gifted and the average as to their analogical transfer.

All verbal problems took the form of riddles, such as the multiplying lilies problem used by Sternberg and Davidson (1983). These were presented as a short story (half a page), ending with instructions to "find the mistake." The cartoons were single-panel cartoons (see Harrison, 1981) accompanied by one verbal statement tinged with humor. Their solution deep structure was analogous to that of the analogous verbal problems. In the two questionnaires that tested for analogical transfer, the surface story of analogous problems was always different, and students did not receive any hints as to a possible connection between the source and target problems.

The rationale for regarding insight problems as problems in every respect, is based on Newell and Simon (1972); Weisberg and Alba (1981); Perkins (1981) and others, who regard insight problems as problems that involve a cognitive solution process that can be analyzed in terms of information processing. The justification for using cartoons as analogous problems to insight problems is based on McGhee (1979). He claims that the cognitive strategies employed in the solution of verbal mathematical problems, non-humoristic riddles that include incongruities, and humoristic stimuli including cartoons, are problems in every aspect, and each one of them can incorporate a similar deep structure.

3.3. Analysis of the solution process from students' protocols

Following each solution, students answered a written questionnaire in which they were asked to reflect on the solution process. The questionnaire was based on a pilot study, with a different group of students, in which a think aloud technique was used to follow students' solution process. Though in the pilot study students were trained to think aloud, it seemed to be too demanding and we decided to switch to post solution protocols. These protocols were collected by written questionnaires that were based on the information obtained in the pilot. The validity of post-solution protocols is supported by the works of Siegler (1989) and Geary and Brown (1991), who claimed that students are capable of describing the strategies they employed, even in a post-solution protocol. Because reports were provided after solving the problem, it is impossible to deduce the chronological sequence of the sub-processes.

Some of the items in the questionnaire were: "Tell in your own words about the story you just read", "Tell in your own words what the problem is in the story", "What are the relevant details for the solution?", "What is your solution?", "Does this problem (or cartoon) remind you of another problem (cartoon)?" Students' answers were used to analyze the sub-processes involved in the solution.

A list of sub-processes and possible categories (of selectivity) for the analysis of students' protocols were defined a priori. If more than one category seemed to be appropriate, the most dominant one was chosen. For each student, only one category for each sub-process (facet) was assigned, leading to one solution profile for each student in each problem solution. The analysis of the responses to the questionnaires was performed by two research assistants, who analyzed separately according to a priori established criteria. The analyses of the two research assistants were compared on a random sample of 17% of all the questionnaires and an agreement rate of 87% was found.

On the basis of the analyzed information, the following variables were defined (note that each student had to solve four problems — two in each learning situation):

3.3.1. Mean Success Score

The solution to each problem was graded on the basis of its correctness; a correct solution scored 3 points, a partially correct one received 2, and an erroneous one received 1 point. A mean score was then calculated for each student, per learning situation.

3.3.2. Mean Selectivity Score

We counted the number of selective sub-processes used by each student. Possible scores on each problem thus ranged between a maximum score of 5 and a minimum of 0. A mean score for each learning situation was then calculated for each solver.

3.3.3. Number of MSPs (Most Selective Profiles)

For each solver, we counted the number of problems in which the most selective levels were employed for *all* the sub-processes (see definition of MSP above). The possible number of MSPs ranged from a maximum of 4 to a minimum of 0.

4. Results

Before examining how well the model captures variance in the solution pathways of different populations, we re-established that the problems used did, indeed, distinguish between gifted and average students on the basis of numbers of successful solutions. Once this was affirmed, we also examined how well selectivity in the sub-processes (MSP) distinguishes between degrees of correctness of the solution, as well as between gifted and average students. Finally, we addressed the question of whether gifted and average students use similar or different solution processes.

4.1. Do the problems really distinguish between gifted and average?

A comparison of Mean Success Scores among gifted and average students (Table 1), as well as the distribution of solutions according to degree of correctness (Table

Table 1
Mean Success Score for gifted and average, before and after learning (s.d. in parentheses)

Learning Situation	Gifted (n=60)	Average (n=55)	Total (n=115)
Situation 1 (no learning)	2.29 (0.85)	1.45 (0.63)	1.89 (0.86)
Situation 2 (after learning)	2.83 (0.57)	2.13 (0.94)	2.49 (0.84)
Total	2.70 (0.36)	1.91 (0.54)	2.32 (0.60)

2), established the capacity of the problems to distinguish between populations. A two-way ANOVA, with repeated measures on the “learning situation” factor, revealed a main effect for giftedness (between-subjects factor) ($F[1,113]=53.90$; $p<0.0001$). That is, in each learning situation (with and without analogical learning) and across situations, the gifted received a higher Mean Success Score than the average students. (A main effect was also found for the learning situation ($F[1,113]=40.37$; $p<0.0001$), signifying that both populations had a higher Mean Success Score after analogical learning than before it.) These findings were also supported by the results in Table 2, which indicate that the gifted are more successful in solving the insight problems used in this research: they are responsible for 70% of all the correct solutions, while the rate for the average is only 30%. Complementarily, the average gave 82% of the erroneous answers, whereas the gifted gave only 18%. A significant relation was found between the correctness of the solution and the type of population (gifted/average) (Chi sq.=84.81; $df=2$; $p<0.0001$). These results confirm that the research problems are capable of capturing the excellent performance of gifted as compared to average, and that this power holds beyond specific learning situations.

Table 2
Distribution of Solutions by Correctness, Gifted and Average

Correctness of Solution	Gifted	Average	Total
Correct	175[a] (70%[b]; 77%[c])	75 (30%; 38%)	250 (100%; 59%)
Partially Correct	34 (49%; 15%)	36 (51%; 18%)	70 (100%; 16%)
Erroneous	18 (18%; 8%)	88 (82%; 44%)	106 (100%; 25%)
Total	227 (53%; 100%)	199 (47%; 100%)	426 (100%; 100%)

[a] Number of solutions
[b] Percentage of total in row
[c] Percentage of total in column

4.2. Does selectivity distinguish between degrees of correctness?

The connection between the selective nature of the sub-processes and the correctness of the solution is established in Table 3. As the table shows, about 86% of the correct solutions are characterized by the use of 4–5 selective sub-processes (out of a maximum of five), whereas only about 3% are characterized by the use of only one selective sub-process or none at all. An opposite picture is obtained in respect to erroneous solutions: about 75% of them included 0–1 selective sub-processes and only 2% included four selective ones. (None involved all five selective sub-processes). Among the partially correct solutions, about 70% included 2–4 selective sub-processes, and the extremes (0–1 or five selective sub-processes) are in the minority. Similarly, the higher the number of selective sub-processes, the greater the percentages of success: out of all solution processes that include five selective sub-processes, about 95% ended with a correct solution, and, out of those that include four selective sub-processes, about 76% led to a correct solution. An opposite trend was obtained for non-selective solution processes: about 83% of the solutions in which none of the sub-processes were selective and about 79% of those with one selective sub-process, were erroneous.

These findings were supported by various statistical tests. The relation between number of selective sub-processes and correctness of the solution was significant (Chi sq.=328.23; df=10; $p<0.0001$), a finding which was also confirmed by one-way ANOVA (see later analysis and Table 5, below). The differences between the levels of selectivity employed in solutions that lead to different degrees of correctness were also significant ($F[2,42]=394.42$; $p<0.0001$). Finally, post hoc multiple comparisons (LSD test) revealed significantly different selectivity between partially correct solutions, and both correct solutions and erroneous ones, as well as between correct and erroneous solutions.

Table 3
Frequency of Selective Sub-processes by Correctness

No. of Selective Sub-processes	Correct Solution	Partially Correct	Erroneous	Total
0	2[a] (5.7%[b]; 0.8%[c])	4 (11.4%; 5.7%}	29 (82.9%; 27.4%)	35 (100%; 8.2%)
1	5 (7.7%; 2.0%)	9 (13.8%; 12.9%)	51 (78.5%; 48.1%}	65 (100%; 15.3%)
2	7 (17.1%; 2.8%)	13 (31.7%; 18.6%)	21 (51.2%; 19.8%)	41 (100%; 9.6%)
3	20 (50.0%; 8%)	16 (40.0%; 22.9%)	4 (10.0%; 3.8%)	40 (100%; 9.4%)
4	68 (76.4%; 27.2%)	20 (22.5%; 28.6%)	1 (1.1%; 2.0%)	89 (100%; 20.9%)
5	147 (94.9%; 59.2%)	8 (5.1%; 11.4%)	0	156 (100%; 36.6%)
Total	250 (58.7%; 100%)	70 (16.4%; 100%)	106 (24.9%; 100%)	426 (100%; 100%)

[a] Number of solutions
[b] Percentage of total in row
[c] Percentage of total in column

Table 4
Distribution of Most Selective Profiles (MSPs) in solutions of gifted and average

No. of MSPs	Gifted	Average	Total
0	3[a] (7.7%[b]; 5.0%[c])	36 (92.3%; 65.5%)	39 (100%; 33.9%)
1	16 (57.1%; 26.7%)	12 (42.9%; 21.8%)	28 (100%; 24.3%)
2	20 (76.9%; 33.3%)	6 (23.1%; 10.9%)	26 (100%; 22.6%)
3	11 (91.7%; 18.3%)	1 (8.3%; 1.8%)	12 (100%; 10.4%)
4	10 (100%; 16.7%)	0	10 (100%; 8.7%)
Total	60 (52.2%; 100%)	55 (47.8%; 100%)	115 (100%; 100%)

[a] Number of students
[b] Percentage of total in row
[c] Percentage of total in column

4.3. Does selectivity distinguish between gifted and average students?

The model claims to differentiate between the populations of gifted and average students, not only in regard to outcome (correct/erroneous solutions), but also in regard to selectivity level in the cognitive sub-processes. This was validated by the Number of MSPs, which was higher for the gifted than for average solvers (Table 4). All solvers (100%) with 4 MSPs (the maximum, as each subject solved four problems) were gifted, whereas about 92% of the solvers with 0 MSPs were average students. Further, an increase in the Number of MSPs is associated with an increase in the percentage of the gifted, and a decrease in the percentage of the average. Finally, t-test analysis for independent samples (gifted/average) revealed a significant difference (t=9.03; df=113; $p<0.0001$) between the mean Number of MSPs among the gifted (Mean=2.15; sd=1.15; n=60) and that among the average (Mean=0.49; sd=.77; n=55).

The connection between giftedness and selectivity in the solution processes is evident. However, the question arises as to whether the observed selectivity results from giftedness or is an artifact deriving from the higher percentage of correct solutions among the gifted. Results point to the former interpretation. Table 5 displays the Mean Selectivity Score of gifted and average students in terms of the correctness of the solution. A two-way ANOVA found a main effect of giftedness ($F(1,420)=56.32$; $p<.0001$), signifying that the processes employed by the gifted are

Table 5
Mean Selectivity Score by Correctness of Solution, gifted and average (s.d. in parentheses)

Correctness of Solution	Gifted	Average	Total
Correct	4.62 (0.67) n=175	3.77 (1.3) n=75	4.36 (0.98) n=250
Partially Correct	3.35 (1.47) n=34	2.47 (1.21) n=36	2.90 (1.41) n=70
Erroneous	1.44 (0.78) n=18	0.94 (0.84) n=88	1.03 (0.84) n=106
Total	4.18 (1.25) n=227	2.29 (1.68) n=199	3.29 (1.74) n=426

significantly more selective than those of the average in each degree of correctness and beyond the learning situation (with or without analogical transfer). It also found a main effect of the correctness of the solution ($F[2,420]=273.20$; $p<0.0001$), meaning that processes ending with a correct solution have a higher degree of selectivity, regardless of whether the solver is gifted or average. No interaction was found between giftedness and solution correctness. This indicates that selectivity is a characteristic of giftedness and not just a characteristic of the correctness of the solution.

4.4. Do the gifted and average use similar cognitive processes to solve problems?

Having established the capacity of the model to distinguish between gifted and average students on the basis of their solution sub-processes, we wish to analyze which of these sub-processes contribute the most to correct solutions for each population. Table 6 presents the results of a stepwise multiple regression analysis of the sub-processes, using the most selective category for each one. It indicates that the nature of the contributing sub-processes differs for the two populations and for the two learning situations.

For the gifted, the sub-processes of selective Combination and selective Encoding are essential in both situations, i.e., the gifted reported that they employed similar processes, albeit at different strengths. Before learning, selective Combination is the strongest sub-process and selective Encoding is second, whereas after learning (situation two) selective Encoding gets priority, and selective Combination is second. A different picture emerges in regard to average students: before learning, the strongest contributing sub-process is selective Retrieval, followed by selective Combination; after learning, it is selective Encoding, followed by selective Retrieval, and

Table 6
Stepwise Multiple Regression Analysis of sub-processes for gifted and average, before and after learning

	Selective Sub-process	R	R^2	Adj. R^2	R^2 change	Sig. F change
Gifted in Situation 1 (no learning)						
Step 1	***C: Combination***	0.882	0.779	0.775	0.779	0.0000
Step 2	*A: Encoding*	0.901	0.812	0.806	0.034	0.0002
Average in Situation 1						
Step 1	***B: Retrieval***	0.661	0.437	0.426	0.437	0.0000
Step 2	*C: Combination*	0.721	0.519	0.501	0.083	0.0004
Gifted in Situation 2 (after learning)						
Step 1	***A: Encoding***	0.965	0.931	0.929	0.931	0.0000
Step 2	*C: Combination*	0.969	0.939	0.937	0.008	0.0007
Average in Situation 2						
Step 1	***A: Encoding***	0.866	0.750	0.745	0.750	0.0000
Step 2	*B: Retrieval*	0.903	0.815	0.808	0.065	0.0000
Step 3	*E: Comparison*	0.913	0.835	0.825	0.020	0.0178

then selective Comparison. The pattern of the sub-processes among the average is completely different from that of the gifted, and it changes after learning.

5. Discussion

This paper presents a model that captures the unique qualities of the solution processes of gifted and average students, going beyond a simple distinction by final outcome. Integrating research findings on gifted vs average, with findings that differentiate between experts and novices, the model provides a generalized framework of comparison, incorporating the sub-processes of Encoding, Retrieval, Combination, Goal Directness and Comparison. Each sub-process is categorized by varying levels of selectivity, with the assumption that the higher the selectivity, the greater the success of the solver and the greater the efficiency and sophistication of understanding.

The model's capability to capture the differing nature of the solution process among gifted and average populations, before and after learning, was established in various ways. Firstly, selectivity was found to be an appropriate measure to distinguish between degrees of correctness of the solution. Secondly, the gifted were found to employ more selectivity in problem-solving, having greater numbers of MSPs (Most Selective Profile). Thirdly, each population used a different pattern of selective sub-processes to arrive at a correct solution; moreover, the gifted use the same sub-processes before and after learning, whereas the average use a different pattern after learning.

The nature of the differing sub-processes employed by the gifted and the average reflects qualitative as well as quantitative differences between populations. The gifted concentrate their efforts on sub-processes that are anchored in the problem to be solved, i.e., selective Combination and selective Encoding. In contrast, average students seem to focus on sub-processes that are more related to their past learning, such as selective Retrieval and selective Comparison. After learning, though successful solutions by both populations are explained by selective Encoding, the average use the supplementary processes of selective Retrieval and selective Comparison, whereas gifted continue to use selective combination.

These results refine the findings of Davidson and Sternberg (1984), who attributed the success of the gifted (as compared to average) to their use of the selective sub-processes of Encoding, Combination and Comparison. The present findings suggest that the difference between the populations can be attributed to different sub-processes, and not just to selectivity in the same ones. Thus, the fact that the success of the gifted is explained mainly by two sub-processes (Encoding and Combination) and that of the average by two (Retrieval and Combination before learning) or three (Encoding, Retrieval and Comparison after learning) supports the findings in the literature of experts/novices (e.g., Berger & Wilde, 1988) and that in the literature on gifted/average (e.g., Davidson & Sternberg (1984)), that pointed to the importance of this process.

It is interesting to note that the sub-process of Goal Directness was not found to contribute to the correctness of solutions. At this stage in our research, we cannot determine whether this stems from the unimportance of this sub-process for correct solutions or from methodological problems. Goal Directness is a central issue in the literature of both experts vs novices (e.g., Lawson & Chinnappan, 1994) and gifted vs average (e.g., Tarshis & Shore, 1991). Further research with different kinds of problems and/or different solvers may shed more light on this issue.

The power of the proposed model stems from its ability to highlight those sub-processes employed in the solution process. Thus, its potential lies not only in being used as a research tool, but also as a useful didactic instrument in assessment and instruction in problem solving. It provides the teacher and the learner with a detailed analysis and understanding of the solution process and highlights points of strength and weakness that can be attended to.

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Appendix

An example of a verbal problem and its analysis is provided. The analogous problem was in the cartoon modality and depicted a big cave man attempting to “rob” the game from two hunters, Mr. Miser and his colleague.

The Rolls and the Boys

Oded, Roni and Yair entered a restaurant. Oded ordered 5 rolls, Roni ordered 4 and Yair the youngest, ordered 3. Each of them paid for the rolls he ordered and they sat down to eat.

Suddenly their cousin Yuval arrived and they invited him to join them and share the rolls equally among themselves. Yuval agreed and the rolls were divided equally among the four of them. When they finished eating Yuval gave them 12 shkalim for his share of rolls that he ate, and ran away. The children were puzzled with the question how to divide the money among them.

Oded suggested dividing the 12 shkalim equally among the three of them, meaning, each one will get 4 shkalim. Roni, on the other hand, suggested that each one should receive a number of shkalim that will equal the number of rolls he paid for, meaning Oded - 5 shkalim Roni - 4 shkalim and Yair, the youngest, should receive 3 shkalim.

They started to argue among themselves, each one claiming that his way is the correct one. Suddenly Yair, who did not interfere in the argument till now, stood up and said: “Dear brothers, both of you are wrong!” The two brothers were deeply involved in the argument stopped arguing, and after a moment started laughing: ”You are right our brother, you are right!”

What was the mistake of the two brothers and what was its origin? What in your opinion, was the suggestion of Yair, the youngest?

The facet (The cognitive sub-process)	The level of selectivity assigned — the feature within the facet	Examples of statements by students
A: Encoding	1. Deep-structure	Three brothers bought a different number of rolls (3,4,5) and contributed, each one, a different number to their cousin so that each of the four will have an equal number of rolls (3). He ate with them and gave them money for the rolls he ate (12). The argument among them stopped when they understood how they have to divide the money in a fair way.
	2. Deep- & surface-structure	Oded, Roni and Yair ordered rolls: Oded paid for 5, Roni for 4 and Yair for 3. When their cousin Yuval arrived they shared with him equally the rolls. When Yuval left he gave them 12 shkalim for the rolls he ate and left them arguing how to divide among themselves the money in the most just way. At the beginning Oded and Roni argued among themselves how to divide the money: should it be equally, or should each one receive asending to the number of rolls he bought? At the end Yair, the youngest, suggested the correct solution.
	3. Surface-structure	Three brothers in a restaurant. When they saw their cousin passing there occasionally, they invited him to eat with them at the restaurant. At the beginning he agreed, and they shared with him the rolls they bought, and then, when he left, they argued among themselves how to divide the money he gave them.

The facet (The cognitive sub-process)	The level of selectivity assigned — the feature within the facet	Examples of statements by students
B: Retrieval	1. Deep-structure	Each one donated a different number of rolls to the cousin so that each of the four will eat an equal number of rolls.
	2. Surface-structure	They did not think about being brothers and quarreled as small children, spitefully. The youngest behaved as a grown up.
C: Combination	1. Integrative	If each one donates a different number, the each one should also receive a different number. Meaning, Oded donated 2 so he should get twice more than Roni who donated only one, and Yair did not donate, as he should get nothing (An integrative correct solution). If each one ate equally, then each one can also receive equally. Meaning, if they had altogether 12 rolls and they shared them, each one received 3 (12:3). Thus they can divide the money amongst themselves, as Roni suggested, and each one will receive a third of the 12 shkalim (An integrative erroneous solution).
	2. Replicative	The solution suggested by Oded to divide the money according to the number of rolls is not correct. Rony’s solution to divide the money equally amongst them is also not correct, thus it seems that only Yair’s solution is probably correct, and both Oded and Roni were convinced.
	3. Distortive	Roni and Oded were wrong because they just saw the money and immediately started arguing. Yair proved them that there is no need to quarrel and a solution can be achieved through consultation. Thus they also started laughing. What is the point of one shekel more or less amongst brothers?

The facet (The cognitive sub-process)	The level of selectivity assigned — the feature within the facet	Examples of statements by students
D: Goal Directness	1. Directed to the final goal	The sum should be divided according to the number of shkalim each one paid for the three rolls that Yuval ate. Oded paid for 2, Roni paid for one and Yair did not pay. Thus the shkalim they received from Yuval should be divided, for Oded twice (8) than for Roni (4), and Yair gets nothing.
	2. Proceeds by systematic search	It should not be divided equally because Oded paid the most nor, should it be divided according to the number of rolls, as finally all ate the same. So if they want a just solution maybe they should take the price of a single role, and then calculate how much each paid for the rolls he bought and give him for each role he did not eat. If, for instance, Oded bought 5 rolls for a certain amount, and finally ate only 3, he should get back the price of two rolls. And if Roni bought 4 rolls but at the end ate only 3, he should receive the price of 1 role. But I don't know the price of one role so I don't think I can reach a solution now.
	3. Proceeds by random search	It should be divided either equally, or according to the number rolls they bought. If the solutions of Oded and Roni are wrong, then maybe Yair's solution is to go to the parents and give them the money and let them solve the problem
E: Comparison		Like in the cartoon with the miser [this refers to the analogous problem in the cartoon modality] because, there too, it is an issue of fair division, when one donates more and one donates less, or donates nothing, as in the case of Yair. Like in the cartoon with the ugly guy with the rod and the two midgets, because the midgets don't have any choice and they have to give him their game, so the best solution is to share the game with him equally…despite the fact that each one caught a different number of geese,

The facet (The cognitive sub-process)	The level of selectivity assigned — the feature within the facet	Examples of statements by students
		and the loss of each one is different. Finally only Mr. Miser is donating because 6:3=2, so only he has to give the ugly one 2 and also to his friend who has only 1 and has nothing to donate. Like in the problem with the hunters, because in both problems it is written about equal sharing. The problem does not remind me of any other problem.
F: Correctness of Solution		The 12 shkalim should be divided between Roni and Oded who donated rolls to Yuval. Nothing is owed to Yair as he did not donate. As Oded donated twice that of Roni he should also receive twice as much. Oded = 8, Roni=4, Yair=0. They have to divide the money amongst the three according to the number of rolls that each one gave to Yuval (no numerous solution). They have to return the money to their cousin Yuval as it is not nice to take money from a cousin.

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